MVT, Optimization, L'Hopital's rule and Integrals

November 11, 2016

Problems

Problem 1. Suppose $0 < f'(x) < \frac{1}{2}$ for all x-values. Show that $f(-1) < f(1) < 1 + f(-1)$.

Problem 2. Sketch the graph of $xe^{1/x}$.

Problem 3. An open-topped cylindrical pot is to have volume 250 cm^3.The material for the bottom of the pot costs 4 cents per cm^2; that for its curved side costs 2 cents per cm^2. What dimensions will minimize the total cost of this pot?

Problem 4. Compute $\lim_{x\to 0}\left(\frac{1}{\sin^2(x)}-\frac{1}{x^2}\right)$.

Problem 5. Compute the following integrals:

1. $\int xe^{x^2}dx$
2. $\int \frac{x+1}{2x-3}dx$
3. $\int \frac{1}{x^2+4}dx$

Problem 6. At time $t=0$ a car is moving at 6 m/s and driver smoothly accelerates so that the acceleration after t seconds is $a(t)=3t$ m/s^2.

1. Write a formula for the speed $v(t)$ of the car after t seconds.

2. How far did the car travel between during the time it took to accelerate from 6 m/s to 30 m/s?